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Transient Error Data Analysis

Stephen R. McConnel Daniel P. Siewiorek Michael M. Tsao

May 1979

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DEPARTMENT of COMPUTER SCIENCE

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Departments of Electrical Engineering
and Computer Science

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Abstract

Experimental data on transient faults from several digital computer systems are presented and analyzed. This research is significant because earlier work on validation of reliability models has concentrated only on permanent faults. The systems for which data have been collected are the DEC PDP-10 series computers, the Cm* multiprocessor, and the C.vmp fault tolerant microprocessor. Current results show that transient faults do not occur with constant failure rates as has been commonly assumed. Instead, the data for all three systems indicate Weibull distributions with decreasing failure rates.

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1. Introduction

The problems posed by transient faults have largely been neglected in the research literature, except for occasional items of limited scope [Wake75]. A general model for transient fault occurrence and recovery based on a Markov model with the assumed exponential distribution for interarrival times was presented in [Aviz77]. This approach is typical of efforts to model all types of faults -- permanent [Depa74] and Intermittent [Spii77, Su78] as well as transient. One goal of the research reported here was an attempt to test the assumed exponential distribution for transient faults.

1.1 Definitions

The following terms need to be precisely defined:

Fault erroneous state of hardware due either to failures of components or to

physical interference from the environment;

Error manifestation of a fault within a program or data structure;

Permanent fault or error which is continuous and stable, reflecting an irreversible

physical change in the hardware;

Intermittent fault or error which is only occasionally detectable, due to unstable

physical failures or environmental conditions;

Transient fault or error due to temporary environmental conditions.

The distinction between intermittent and transient faults is not always made in the literature [Kama75, Tasa77]. The dividing line chosen for this study is that of the applicability of testing [Breu73, Kama74, Losq78, Savi78]. Faults due to underlying physical conditions (permanent and intermittent) of the hardware or unstable but repeated environmental conditions are at least potentially detectable by testing, and then repairable. However, faults due to temporary environmental conditions are incapable of repair, as the hardware is physically undamaged. It is this attribute of transient faults which magnifies their importance. Even in the absence of all physical defects, including those manifested as intermittent errors, faults will still occur. Fault tolerant techniques will still be required to prevent such errors from causing computer systems to crash.

1.2 Significance of Transient Errors

The importance of transient faults comes from two factors: their relatively frequent

occurrence and the impossibility of preventative maintenance. Several studies have shown that permanent faults cause only a small fraction of all detected errors. The available figures for several systems are given in Table 1-1 below [Full78, Morg78, Morg78a, Slew78]. The last row of figures are estimates comparing the hard and soft failure rates for a one megaword by 37 bit memory array composed of 4K MOS RAMs [Geil79, Ohm79]. (Soft failures are transient failures caused by the radioactivity in the packaging of the chips themselves.)

	Detection	MTBE per	MTTF per	
System Technology	<u>Mechanism</u>	Processor	Processor	MTBE/MTTF
CMUA PDP-10, ECL	Parity	44 hrs.	800-1600 hrs.	0.03-0.06
Cm + LSI-11, NMOS	Diagnostics	128 hrs.	4200 hrs.	0.03
C.vmp TMR LSI-11	Crash	97-328 hrs.	4900 hrs.	0.02-0.07
Telettra, TTL	Mismatch	80-170 hrs.	1300 hrs.	0.06-0.13
1M x 37 RAM, MOS	(Parity)	106 hrs.	1450 hrs.	0.07

Table 1-1: Ratios of Transient to Permanent Errors

1.3 Overview of Paper

Section 2 describes the three different architectures for which data has been gathered, and gives details about the software systems used on two of them for semi-automated data collection. The data analysis in Section 3 shows how a decreasing failure rate Welbuil process fits the data better than a Poisson process. General conclusions and areas for further research are detailed in Section 4.

2. Data Collection for Transient Errors

2.1 Description of Systems

Short descriptions of the three architectures for which data has been gathered are given to show the wide range of system types which all support the same general conclusion of decreasing failure rates.

2.1.1 Description of PDP-10 Systems

The PDP-10¹ is a general purpose 36-bit computer packaged either as a DECsystem-10 running a time sharing operating system called TOPS-10, or as a DECsystem-20 running TOPS-20 [Bell78]. The main system for the Department of Computer Science at Carnegie-Mellon University is a DECsystem-10 known as CMUA, with a KL-10 (ECL) processor, one megaword of memory, eight disk drives totalling 1600 megabytes of online storage, and two magnetic tape drives. In addition, the system is connected to a PDP-11 front end processor which multiplexes a large number of video terminals. This system is used almost exclusively to support the research programs of the department.

Other systems for which data has been collected in this study are the three DECsystem-20's operated by the university's Computation Center. These systems, known as TOPSA, TOPSB, and TOPSC, are used to support general educational and administrative needs of the university. TOPSA, used primarily for administrative work, has 256K of memory, four disk drives totalling over 700 megabytes of online storage, and two magnetic tape drives. TOPSB and TOPSC, used for educational support, each have 512K of memory, three disk drives with 528 megabytes of online storage, and two magnetic tape drives. Each of these three systems also connects to a number of terminals.

2.1.2 Description of Cm*

The structure of Cm* is shown in Figure 2-1. This is a structure with a low concurrency switch (the network of buses) giving access to a shared memory. The structure is built from Processor-Memory pairs called Computer Modules or Cm's. The memory local to a processor is also the shared memory of the system. Each Cm has a local switch, or Slocal, which interfaces its bus to the rest of the system. A mapping controller, or Kmap, is shared by several Cm's, which are connected to it by their Slocal's to form a "cluster". Each Kmap in the system is connected via multiple intercluster buses to other Kmaps, completing the

¹DEC, PDP-10, DECsystem-10, TOPS-10, KL-10, DECsystem-20, TOPS-20, PDP-11, and LSI-11 are all registered trademerks of Digital Equipment Corporation.

interconnection scheme. The Kmaps perform all the functions necessary for meeting both intracluster and intercluster memory requests. With this structure, any processor can access any memory in the system, with increasing time penalty for the "distance" away--it's own local memory, memory belonging to another processor in the same cluster, or memory within another cluster.

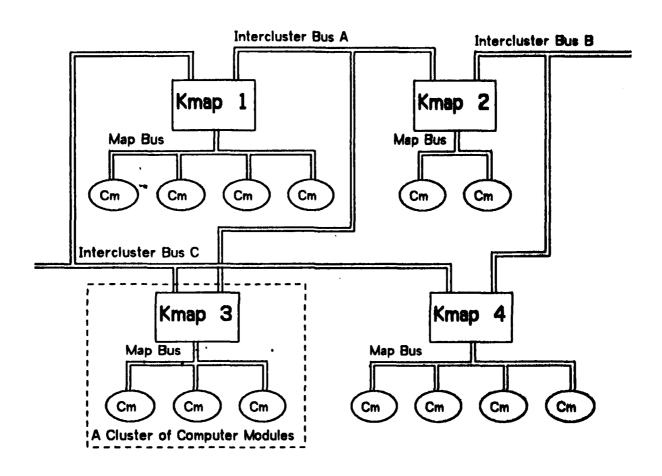


Figure 2-1: Structure of Cm*

Figure 2-2 shows the details of a Computer Module. The processor is a DEC LSI-11. Each Cm has dynamic RAM memory and individual Cm's have a variety of I/O devices. Several Cm's have serial links to the Cm* host computer, a message switching PDP-11 [Scel77], for facilitating user access and program loading. Other devices include disk drives and line time clocks.

The original configuration of Cm*, shown in Figure 2-3, contained ten Cm's connected to three Kmaps to form three clusters, two with four Cm's and one with two. In this setup, referred to as Cm*/10, each Cm had a serial link to the Cm* host. This greatly facilitated the

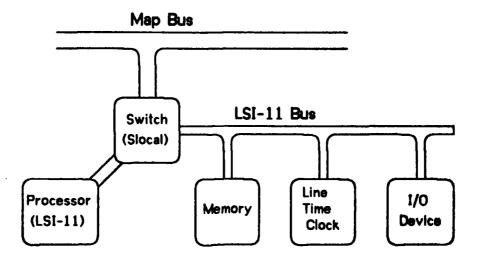


Figure 2-2: Cm* Computer Module

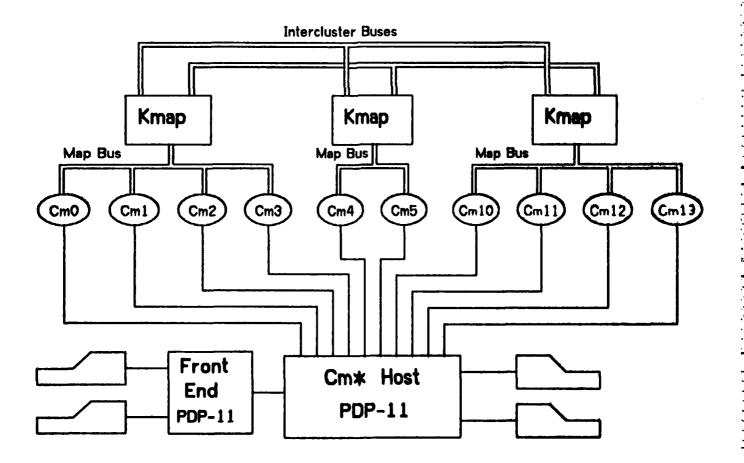


Figure 2-3: Cm*/10 Configuration

initial sytem debugging and software development efforts.

The architecture of Cm* has been widely reported in the literature. See [Swan77, Swan78] for detailed descriptions and additional references to published reports.

2.1.3 Description of C.vmp

C.vmp is a triplicated NMOS LSI-11 microprocessor with voting at the bus level [McCo78, Siew77, Siew78, Siew78a]. There are three processor-memory pairs, each pair connected via a voter circuit shown in Figure 2-4. With the voter active, the three buses are voted upon and the result of the vote is sent out. Any disagreements among the processors will, therefore, not propagate to the memories and vice versa. If the nonredundant portion of the voter represents a system reliability "bottleneck", triplicated voters can be used. With this scheme even the voter can have either a transient or a hard failure and the computer will remain operational. Note that voting is done in parallel on a bit by bit basis. A computer can have a failure on a certain bit in one bus, and proper operation will continue, provided that the other two buses have the correct information for that bit. There are cases, therefore, where failures in all three buses can occur simultaneously and the computer would still be functioning correctly.

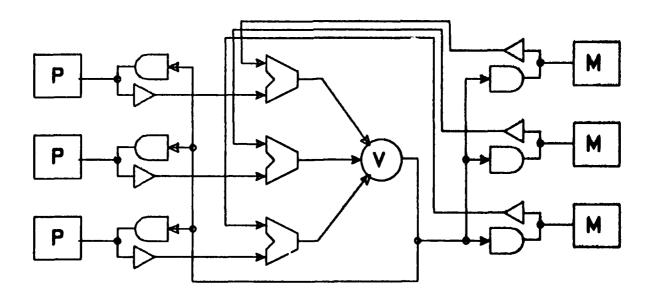


Figure 2-4: C.vmp Voter Circuit

The current implementation of C.vmp contains 12K words of dynamic MOS memory and a dual floppy drive per processor. The configuration and network attachments are shown in Figure 2-5. The data reported was gathered from the detailed operations log kept by the

users. All crashes not traced to hardware failures, software bugs, or operator mistakes were assumed to result from transient faults.

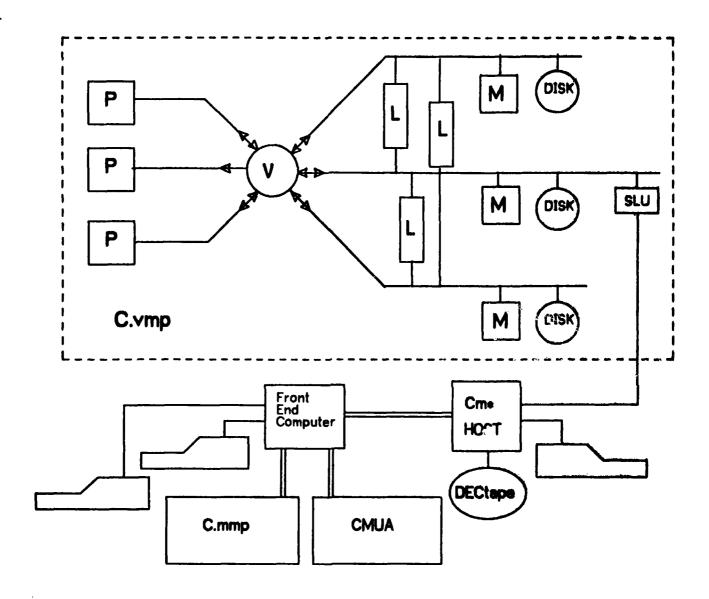


Figure 2-5: C.vmp Configuration

2.2 SEADS: Data Collection on the PDP-10

2.2.1 System Error Files

The core of the PDP-10 error reporting system is the online error log file maintained by the TOPS-10 and TOPS-20 operating systems. Entries are made in this file for a variety of reasons, most notably system reloads and memory and I/O errors [Digi78]. Each entry

contains the date and time at which it was made, the processor serial number, and information about the type of error or other condition being reported.

2.2.2 SEADS Program

To facilitate statistical analysis of transient errors on PDP-10's, a program named SEADS (Statistical Error Analysis Data Summary) has been written which derives interarrival times and time-of-day distributions from the system error log files. The outputs generated include the following:

- Lower bound estimates of system availabilities, in total and for each file processed;
- Graphs of the time-of-day distribution of entries, divided into 48 half-hour segments;
- Graphs of the distributions of interarrival times for all entries in total, for each entry individually, and for arbitrary sets of entries;
- Data files containing the time-of-day distribution and the lists of interarrival times and error types.

Examples of the first three types of outputs are shown in Figures 2-6, 2-7, and 2-8. One feature of Figure 2-8 should be explained. As the data must be scaled to fit on a fixed size histogram, some way of displaying data points which are too small to be shown in scale must be developed. The convention adopted was to use two different characters: 'x' for full scale values, and 'o' for values too small to otherwise be displayed. Please note that while Figure 2-6 is only a sample, the other two figures represent the sum total of all files processed from all four systems, comprising over 8000 hours.

To facilitate data collection, SEADS can read up to 100 separate system error log files. It can also read the data files it generates to restore the internal program data state, thus minimizing the computational requirements for analyzing large numbers of files.

2.3 AUTODIAGNOSTICS: Data Collection on Cm+

2.3.1 Motivation of AUTODIAGNOSTICS

Cm* is an experimental system in which the hardware reliability is always suspect. Those working on software development would benefit from knowing the current hardware status. This is the main purpose of the diagnostic software on Cm*. The test programs can also detect transient errors, as was proposed by [Tasa77], and demonstrated by the

AUTODIAGNOSTICS system. Furthermore, the results from this study can also help Cm² software researchers to judge the effectiveness of error recovery routines used for achieving high availability.

2.3.2 Error Reporting System

2.3.2.1 CMDIAG

Automatic diagnostic software was developed to exercise idle modules in Cm*. One such module was used to run the master control program CMDIAG. This module operated in a special mode which enabled it to control other Cm's as though it was a user at a terminal. With this ability, CMDIAG acquired control of all unassigned modules and continuously cycled each of them through a series of four diagnostic tests. The program was able to dynamically acquire and release modules in response to the changing needs of the other users.

The test programs were designed to halt processor execution whenever an error was detected. CMDIAG detected this situation and interrogated the affected Cm's memory and registers for pertinent information, which was then printed on the Cm* host console terminal:

Location of error

the module that detected the error, the test that was running in that Cm, and the number of passes completed since the test program was loaded into the module.

Time information

the time of day, the total number of module-hours accumulated up to that time, the module-hours accumulated by that particular Cm, and the module-hours accumulated by that test.

Symptoms of the fault

error number, the ID number of the subtest, and the PC and SP values.

2.3.2.2 Test Programs

The sequence of test programs consists of four diagnostic programs. Each is designed to test a certain portion of the Cm* hardware. They are primarily intended to detect hard failures (permanent or intermittent faults), but can be used to detect transient faults as well.

Instruction Test--DVKAA

a processor exerciser which tests for correct operation of all the basic instructions (excluding traps) and all the addressing modes [Digi75a]. Less than one-tenth of a second is required for each successful pass through this test: therefore many hundreds of passes are run before loading the next test program.

Interrupt and Trap Test--DVKAD

a processor exerciser which tests for correct operation of all trap and breakpoint instructions, and for interrupt and nonexistent memory traps

as well [Digi75b]. Each pass through this test takes a few seconds to complete.

Memory Diagnostic--DZKMA

a memory exercisor for 4K to 128K of dynamic MOS RAM on LSI-11's which consists of a series of 13 test routines [Digi75c]. Each pass through this program requires several minutes, depending on the size of the memory being tested.

Slocal Test

a locally written exhaustive test of the Slocal hardware which also tests part of the Kmap hardware by doing a small memory test in "mapped" mode. This program completes a pass within a few minutes.

SEADS VERSION 3A(100) ERROR FILE ANALYSIS

COUNT OF BAD TIME ERRORS:

TOTAL NUMBER OF ENTRIES FOR ALL INPUT FILES: 18445
TIME SPAN: 1542 HRS., FROM: 17-Feb-79 5:83:11 TO: 18-May-79 11:38:59
APPROXIMATE SYSTEM AVAILABILITY: 8.877

SYSTEM #2149 NUMBER OF ENTRIES: 344
TIME SPAN: 170 HRS., FROM: 17-Feb-79 5:03:11 TO: 24-Feb-79 7:30:06
RPPROXIMATE SYSTEM AVAILABILITY: 0.987

SYSTEM #2227 NUMBER OF ENTRIES: 2045
TIME SPAN: 150 HRS., FROM: 24-Feb-79 22:22:00 TO: 3-Mar-79 5:09:58
APPROXIMATE SYSTEM AVAILABILITY: 0.947

SYSTEM #2326 NUMBER OF ENTRIES: 1149
TIME SPAN: 140 HRS., FROM: 3-Mar-79 5:43:84 TO: 9-Mar-79 1:55:27
APPROXIMATE SYSTEM AVAILABILITY: 8.894

SYSTEM #1686 NUMBER OF ENTRIES: 12967
TIME SPAN: 1681 HRS., FROM: 3-Apr-79 16:61:24 TO: 18-May-79 11:36:59
APPROXIMATE SYSTEM AVAILABILITY: 0.847

Figure 2-6: Sample File/Availability Output from SEADS

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MON-RELGORD HONITOR ERROR	X X X X X X X X X X X X X X X X X X X	x	****	*********
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MON-RELICAD HONITOR ERROR				
MON-RELORD HONITOR ERROR				
MON-RELORD HONITOR ERROR		**		
MON-RELOAD MONITOR ERROR		X X X X		
MON-RELOAD MONITOR ERROR				
MON-RELORD MONITOR ERROR	X X X X X X X X X X X X X X X X X X X	X X Y Y	X	
MON-RELOAD MONITOR ERROR	XXX	X	X XX	
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NON-RELOAD MONITOR ERROR	NETHORK LINE STATS	<238> 858		
NON-RELOAD MONITOR ERROR	NETHORK UP-LINE DUMP	<203> 1		
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NON-RELORD MONITOR ERROR	PROCESSOR PARITY TRAP	<160> 35		
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NON-RELOAD MONITOR ERROR <002> 626 CPU NXM ERROR <004> 9766	DISK UNIT ERROR Magtape Statistics			
NON-RELOAD MONITOR ERROR <ab2> 626</ab2>	DATA CHANNEL ERR Disk unit error Magtape Statistics	<886> 7338		
	CPU NXM ERROR Data Channel Err	<004> 9766 <006> 7330		

Figure 2-7: Sample Time of Day Distribution Output from SEADS

DISTRIBUTION OF INTERARRIVAL TIMES

X	MINIMUM VALUE: MEAN TIME: RXIMUM VALUE:	8.88 SEC 18.99 HOURS 183	TIME IS 8.88 SEC. C. MAXIMUM VALUE: STANDARD DEVIATION: SCALE FACTOR:	15.78 HOURS 3	HODE	"BUCKET #":	1 ENTRIES:	241
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Figure 2-8: Sample Interarrival Time Distribution Output from SEADS

3. Data Analysis for Transient Errors

3.1 Analyzing Interarrival Times

3.1.1 Distribution of Interarrival Times

The interarrival data can be plotted as a histogram (as in Figures 2-8 and 3-2) to form an approximation to the probability density function of transient errors. This is useful in deciding initially on which distributions to study. The obvious skews toward the low end for all the data collected on these systems indicate that the Weibull distribution should be used. The exponential distribution is a special case of the general Weibull function.

3.1.2 The Weibull Distribution

A review of the Weibull distribution and its related functions is in order. The Weibull distribution has two parameters: α (the shape parameter) and λ (the scale parameter). (Several formulations Weibull of the function the literature appear in [Barl75, Mali75, Roma77, Shoo68, Thom69]. The most common of these are presented in Appendix 1.) The probability density function (pdf), cumulative distribution function (CDF). reliability function, and hazard (failure rate) function of the Weibull distribution are shown in Equations 1 through 4 ($\alpha > 0$, $\lambda > 0$):

$$pdf = f(t) = \alpha \times \lambda \times (\lambda t)^{\alpha - 1} \times exp[-(\lambda t)^{\alpha}]$$
 (1)

$$CDF = F(t) = 1 - \exp[-(\lambda t)^{\alpha}]$$
 (2)

Reliability =
$$R(t) = \exp[-(\lambda t)^{\alpha}]$$
 (3)

Hazard Function =
$$z(t) = \alpha \times \lambda \times (\lambda t)^{\alpha-1}$$
 (4)

Note that the values of all these functions depends on time only through the product of the scale factor and time $--\lambda t$. Also, the hazard function z(t) shows the effect of the shape parameter on the failure rate:

- if α < 1, the failure rate is decreasing with λt_i
- if $\alpha=1$, the failure rate is constant with time, resulting in an exponential distribution; and

- if $\alpha > 1$, the failure rate is increasing with time.

The goal of this study is to find the value of the shape parameter α for the underlying distribution of transient errors.

When performing statistical analysis of data, two common measures are the mean (μ) and standard deviation (σ). For the Weibull distribution, these are defined as follows in terms of α and λ :

$$\mu = \lambda^{-1} \times \Gamma((\alpha+1)/\alpha) \tag{5}$$

$$\sigma = \lambda^{-1} \times \left[\Gamma((\alpha+2)/\alpha) - \Gamma^2((\alpha+1)/\alpha) \right]^{1/2}$$
 (6)

where the gamma function $\Gamma(\omega) = \int_0^\infty \rho^{\omega-1} \exp(-\rho) d\rho$

The influence of the Weibull parameters on the mean of the distribution is illustrated in Figure 3-1. The maximum likelihood estimates of the Weibull parameters for the recorded data are indicated by diamonds in the graph.

With just these two statistics, the Weibull failure rate can be determined to be decreasing, constant, or increasing as follows:

- if $\mu < \sigma$, the failure rate is decreasing;
- if $\mu = \sigma$, the failure rate is constant;
- if $\mu > \sigma$, the failure rate is increasing.

3.1.3 Estimation of the Weibull Parameters

In the following equations, $\{X_1, X_2, ... X_N\}$ is the set of interarrival times.

The Weibull cumulative distribution function (Equation 2) can be rearranged into the following:

$$Ln Ln [1 / (1 - F(t))] = \alpha \times Ln(t) + \alpha \times Ln(\lambda)$$
(7)

Equation 7 is the basis for using linear regression analysis to fit the data to a Weibull distribution [Berg74, Roma77]. If the data is from a Weibull distribution, its plot should approximate a straight line. The slope of the straight line is an estimate of α , and the Y-intercept divided by the slope is an estimate of Ln (λ). The value of the function F(t) is

estimated by

$$F(t_i) = (j - 0.5) / N$$
 (8)

The maximum likelihood estimators (MLE) α' and λ' for the Weibull distribution satisfy the following equations [Thom69]:

$$(N / \alpha') + \sum_{j=1}^{N} Ln(X_j) = N \times (\sum_{j=1}^{N} X_j^{\alpha'} \times Ln(X_j)) / (\sum_{j=1}^{N} X_j^{\alpha'})$$
 (9)

$$(\lambda')^{\alpha'} = N / \sum_{j=1}^{N} x_j^{\alpha'}$$
 (10)

Once the value of the shape parameter α' , is known, Equation 10 can be used to calculate the scale parameter λ' . Equation 9 can be used to derive a difference equation in the form

$$\alpha'_{i+1}$$
 = Function (α'_i, \vec{X}_N)

A quickly converging solution can be found by using the Newton-Raphson method, as was presented in [Thom69]. The linear estimate of α found by the linear regression analysis described above is useful as an initial value for the iterative solution process.

3.1.4 SEAPLT: Automated Weibull Plot Analysis

In order to facilitate the analysis of large amounts of data, the methods described above were implemented in a program named SEAPLT (System Error Analysis PLoTter). Two separate plotting files are output by this program:

- a Weibull Plot showing the linearized fit of the data to the distribution; and
- an adjusted histogram of the interarrival time distribution, with an approximate Weibull probability density function superimposed.

In addition to the plots, both the linear and MLE estimates of α and λ are calculated, along with the mean and standard deviation of the data. All the graphs used in this paper and all the values given in the tables were generated by this program. The procedures for the linear regression analysis and MLE calculations are presented in Appendix II.

3.2 Graphical Data Analysis

System reloads were chosen as likely to reflect transient errors, as they are commonly caused by the ubiquitous "crash". In systems with stable hardware and matured software, the most frequent cause of crashes appears to be transient errors.

In scanning the data generated by SEADS, it became clear that the PDP-10 systems frequently recorded several errors for one fault. To mask out the effects of this, error

entries within five minutes of a previous entry were counted as a part of the previous fault. It was felt that five minutes was a reasonable threshold for the study. The software ellowed any choice for the threshold, facilitating examination of the the sensitivity of the data to threshold values 1.

Two groups of data are presented for system reloads. The first is taken from the individual system (TOPSC) which had the most complete data. The second is the collected data from all four systems.

Figures 3-2 and 3-4 show histograms of the distributions of the interarrival times for system reloads on TOPSC and for all four systems, overlaid with the MLE Weibull probability density functions. Figures 3-3 and 3-5 show the plots of the TOPSC and overall PDP-10 reload data on Weibull paper. (The straight lines drawn on all the Weibull plots are least squared error fits to the data.) Note that most of the visual deviation is due to relatively few points at the lower end.

The second class of events likely to reflect transient errors in the PDP-10 data was the memory parity error interrupt. Except in the case of failing devices which cause intermittent, and finally permanent, faults, these are always the result of transient faults in the memory system. Figures 3-6 and 3-7 show the interarrival distribution and the Weibull plot of the data. In this case, only the total data for all four systems is shown, as too few data points were collected from any one of the four systems to be statistically significant.

Figures 3-8 and 3-10 shows the adjusted histograms of the interarrivals for Cm* and C.vmp respectively. Figures 3-9 and 3-11 are plots of the interarrival data for each system's transient errors on Weibull paper. The straightness of the data shows that the samples follow a Weibull distribution.

3.3 General Statistics and Confidence Intervals

Table 3-1 lists some general statistics about the interarrival times for the five sets of data: TOPSC reloads, PDP-10 reloads, PDP-10 parity errors, C.vmp crashes, and Cm* transient errors. In all cases, the mean is less than the standard deviation, indicating a decreasing failure rate ($\alpha < 1$).

A 90% confidence interval for α and λ was generated for the last three sets using methods developed in Thoman et al. [Thom69] (The 90% confidence interval is the range within which the actual value of the estimated parameter would fall ninety percent of the time if the

 $^{^{1}}$ Threshold values of one minute and ten minutes were also tried without changing the results presented here.

	TOPSC <u>Reload</u>	PDP-10 Reload	PDP-10 <u>Parity</u>	<u>Çm</u> ≠	Cymp ²
Time (hrs)	2646	8 576	8596	4222	4921
Errors	195	636	74	103	50
Interarrivals	196	640	78	104	51
μ	13.5	13.4	110.2	40.6	96.5 (328)
σ	16.5	24.6	244.9	59.8	167.8 (471)
α (Linear)	0.864	0.684	0.500	0.834	0.711
α' (MLE)	0.826	0.639	0.481	0.779	0.654
λ (Linear)	0.0843	0.109	0.0206	0.0294	0.0149
λ' (MLE)	0.0826	0.106	0.0203	0.0288	0.0146

Table 3-1: Statistics for Transient Errors

experiment was repeated many times.) These values are listed in Table 3-2. Note that the range of values for α does not include 1.0 (i.e., the exponential distribution) for any of the three sets of data.

	PDP-10 Parity	<u>Cm</u> ∗	<u>C.ymp</u>
$[\alpha_{low}, \alpha_{high}]$	[0.421,0.566]	[0.693,0.893]	[0.558,0. 8 06]
$[\lambda_{low}, \lambda_{high}]$	[0.0134,0.0307]	[0.0231,0.0359]	[0.0099,0.0214]

Table 3-2: 90% Confidence Intervals for a and A

3.4 Goodness of Fit Test

To confirm the impression given by the Weibull plots that the data collected on transient errors for the various systems does in fact fit a Weibull distribution, a Chi-square goodness of fit test was made for each of the five sets of data. Basically, such a test divides the data into several equiprobable regions and measures the deviation from the expected number of samples in each of these regions:

O; = observed frequency in the i'th region

²Note that the pessimistic value discussed in [Siew78] is used throughout for C.vmp because there were tes few interarrivals in the optimistic value (shown in perentheses for the mean and standard deviation) to be statistically algorithm.

E; = expected frequency in the i'th region

When the number of samples is large enough (as is the case for all sets of data under consideration), then the following statistic follows a χ^2 distribution with k-3 degrees of freedom:

$$Q = \sum_{i=1}^{k} (O_i - E_i)^2 / E_i$$
 (11)

The results of the tests performed are given in Table 3-3.

	TOPSC Reload	PDP-10 <u>Reload</u>	PDP-10 <u>Parity</u>	<u>Cm</u> ≠	Cvmp
Q	23.36	6.40	6.72	9.46	3.71
Degrees of Freedom d	34	5	11	17	7
Level of Significance p	0.90	0.25	0.80	0.90	0.80
$\chi^2_{p,d}$	23.95	6.63	6.99	10.08	3.82

Table 3-3: X² Goodness of Fit Test Statistics

The level of significance of the Chi-square test is the probability of erroneously rejecting the hypothesis that the data is from the given distribution. (Statisticians call this a "Type I" error [DeGr75, Roma77].) Large values are desirable for this figure if they can be achieved, as the likelihood of wrongly accepting a false hypothesis ("Type II" error) is inversely related to the probability of Type I errors for fixed sample sizes. Typical values for the level of significance in statistical tests range from 0.10 to 0.01. All of the results shown in Table 3-3 are much higher than this, showing very good fits to the Weibull distribution.

To complete the testing procedure, a Chi-square test was done for each of the five sets of data assuming an exponential distribution. The comparison of these results are shown in Table 3-4. Although the exponential hypothesis fits the data fairly well in a couple of cases, in every case the Weibull fit is much better.

	TOPSC Reload	PDP-10 <u>Reload</u>	PDP-10 <u>Parity</u>	<u>Cm</u> ∗	C.ymp
Q	30.61	252.55	79.95	15.14	18.35
Degrees of Freedom d	30	6	12	13	7
Level of Significance p	0.40	0.00	0.00	0.25	0.01
$\chi^2_{\rm p,d}$	31.32	œ	œ	15.98	18.48

Table 3-4: X² Test of Exponential Distribution

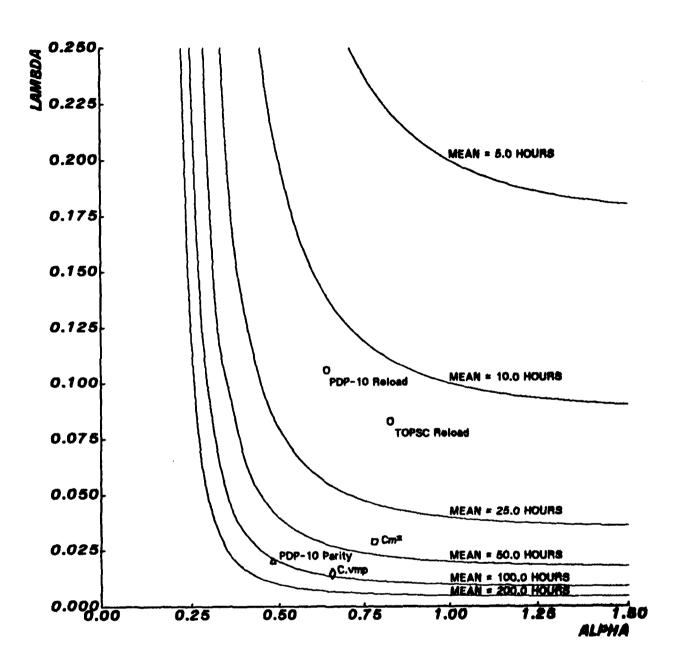


Figure 3-1: Means of Weibull Distributions

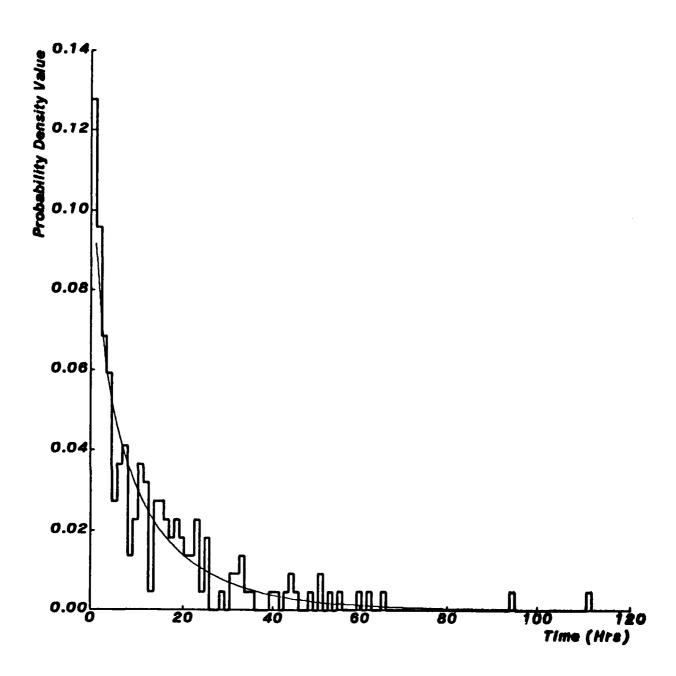


Figure 3-2: Distribution of TOPSC System Reloads

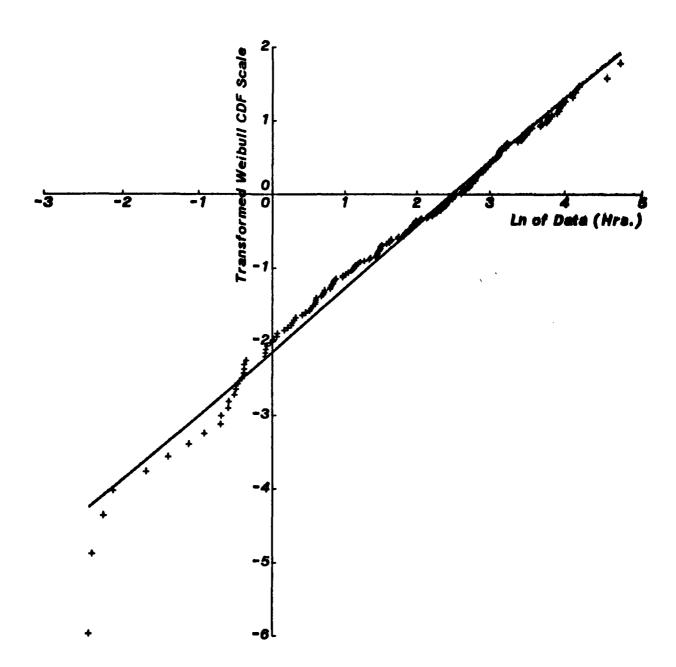


Figure 3-3: Weibull Plot of TOPSC System Reloads

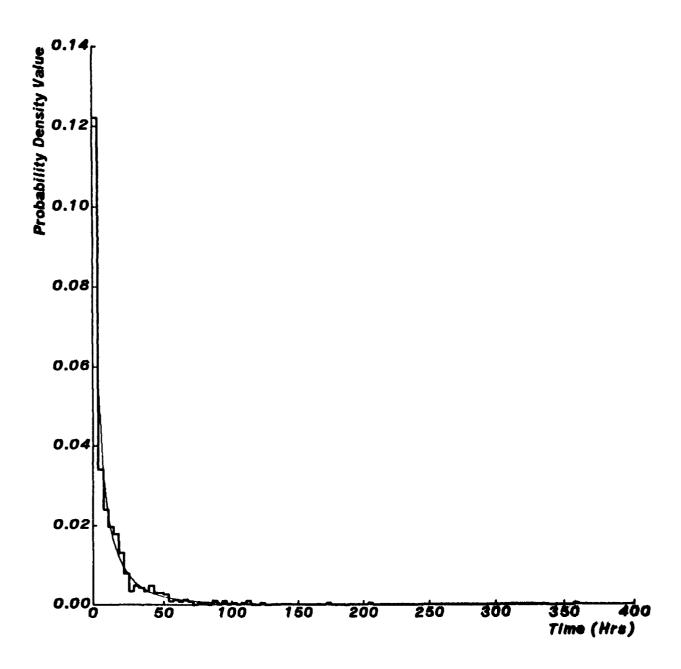


Figure 3-4: Distribution of PDP-10 System Reloads

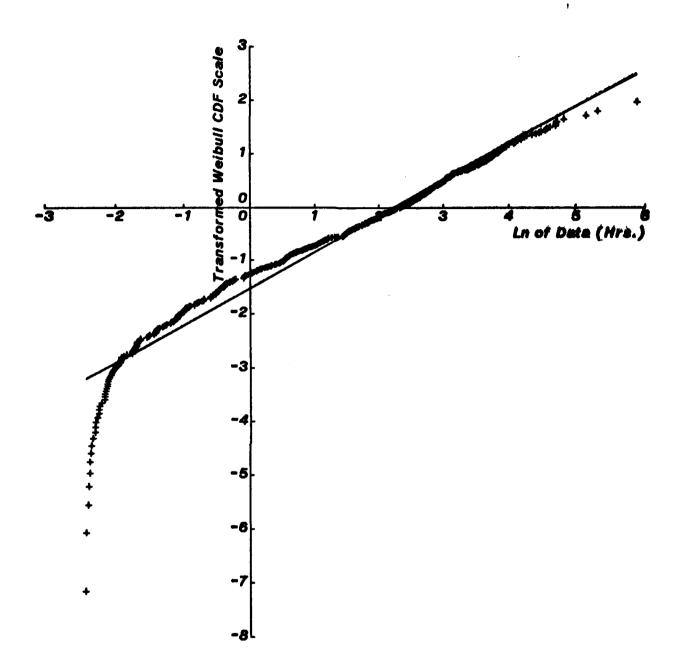


Figure 3-5: Weibull Plot of PDP-10 System Reloads

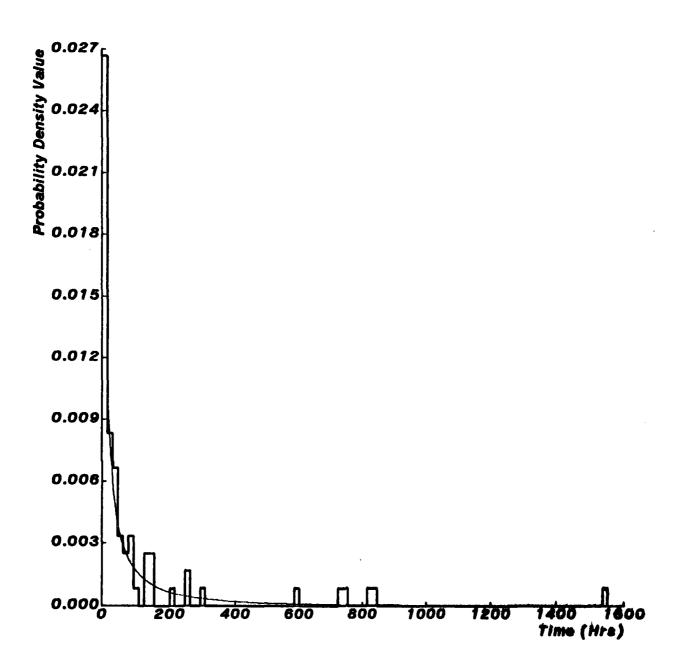


Figure 3-6: Distribution of PDP-10 Parity Interrupts

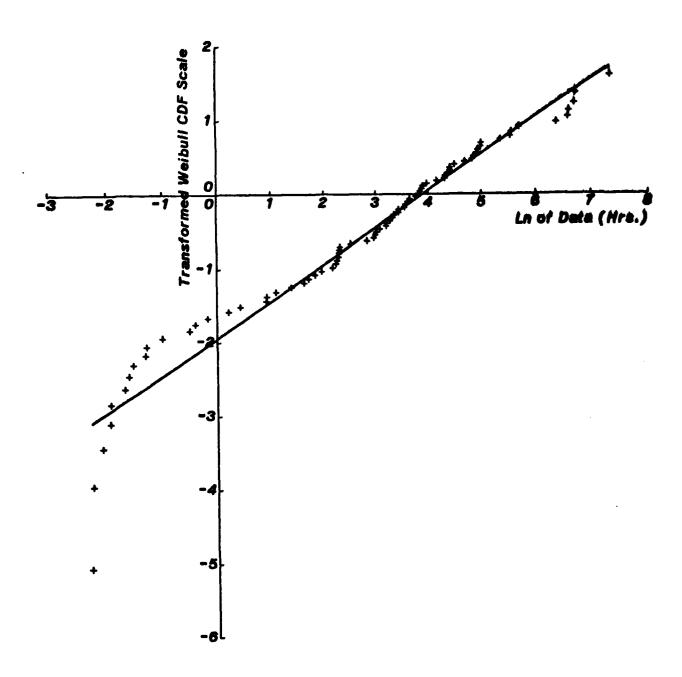


Figure 3-7: Weibull Plot of PDP-10 Parity Interrupts

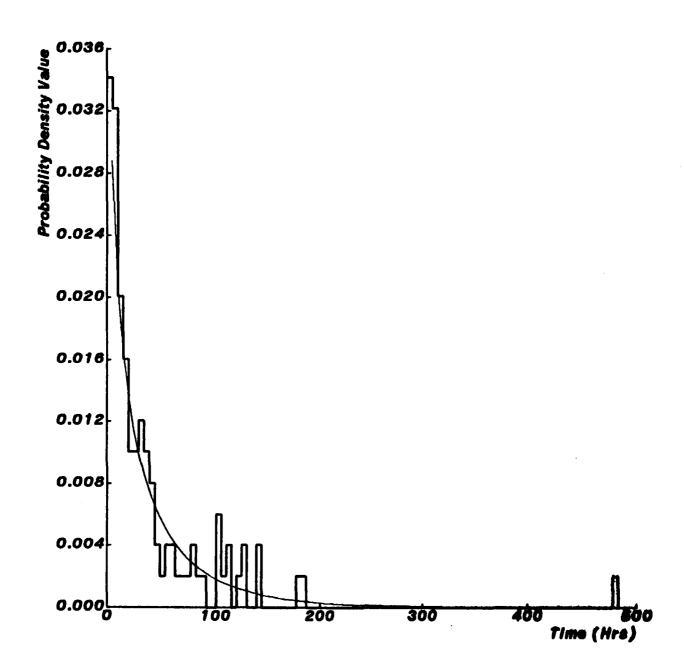


Figure 3-8: Distribution of Cm* Transient Errors

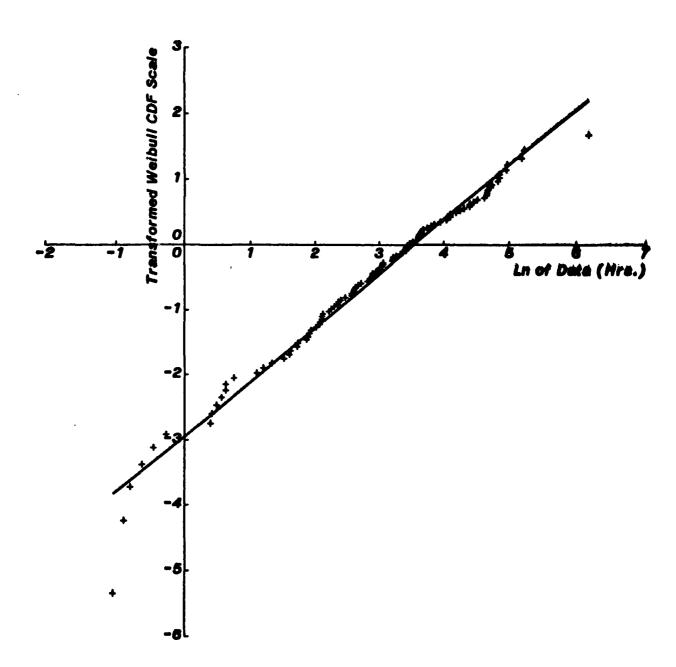


Figure 3-9: Weibull Plot of Cm+ Transient Errors

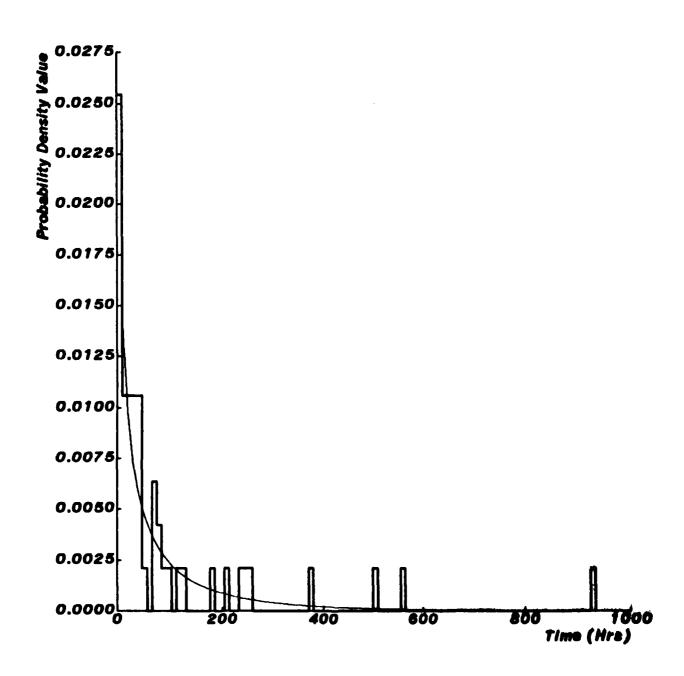


Figure 3-10: Distribution of C.vmp Crashes

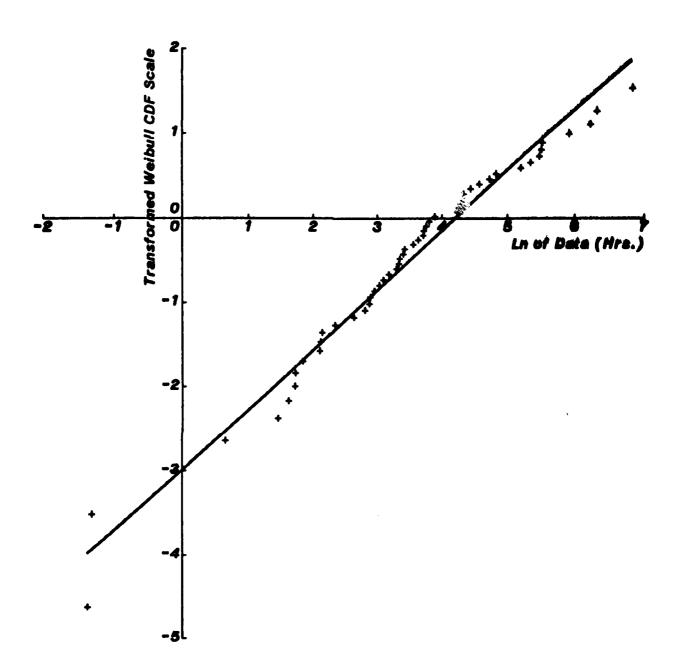


Figure 3-11: Weibull Plot of C.vmp Crashes

4. Conclusions

The crash history of C.vmp from August 1977 to May 1978 (4900 hours) shows that the interarrival times of transient errors follow a Weibull distribution with a shape parameter less than 1, which indicates a decreasing failure rate. Similarly, analysis of transient error data collected on Cm* from September 1977 to August 1978 (4200 hours) and data collected from PDP-10 systems between August 1978 and March 1979 (8600 hours) seem to support the same conclusion. These systems range in size from an NMOS microprocessor with 12K words of memory to ECL mainframes with one megaword of memory, and range in redundancy from nonredundant to parity to triplication. This wide range application of the decreasing failure rate Weibull distribution means that much rethinking will have to be done on transient modelling and statistical analysis of transient data.

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I. Forms of the Weibull Distribution

Five main formulations of the Weibull distribution appear in the literature [Bar175, Mali75, Roma77, Shoo68, Thom69]. The probability density functions for each of these is presented below. Note that the first one presented is the same as that given in Section 3.

$$f(t; \alpha, \lambda) = \alpha \times \lambda^{\alpha} \times t^{\alpha-1} \times \exp[-(\lambda t)^{\alpha}]$$

$$f(t; \alpha, \theta) = \alpha \times \theta \times t^{\alpha-1} \times \exp[-\theta \times t^{\alpha}]$$

$$f(t; \alpha, b) = \alpha \times b^{-\alpha} \times t^{\alpha-1} \times \exp[-(t/b)^{\alpha}]$$

$$f(t; \alpha, \beta) = (\alpha/\beta) \times t^{\alpha-1} \times \exp[-(t^{\alpha})/\beta]$$

$$f(t; m, K) = K \times t^{m} \times \exp[-(K/(m+1)) \times t^{m+1}]$$

Following is a table which gives the equivalent parameter values for the different forms of the Weibull distribution. The first column gives the values of the shape parameter in terms of the value of α used in Section 3. The second column gives the values of the scale parameters in terms of λ from the same formulation. The third column gives the value of λ in terms of the different shape parameters and the fourth column gives the value of λ in terms of the different scale parameters.

	Shape	<u>Scale</u>	•	λ
f(t; α,λ)	α	λ	α	λ
f(t; α,θ)	α	λα	α	g(1/ac)
f(t; α,b)	α	λ-1	α	b ⁻¹
$f(t; \alpha, \beta)$	α	λ-α	•	β-1/ a
f(t; m,K)	α-1	$\alpha \times \lambda^{\alpha}$	m+1	(K/(m+1)) ¹ /(m+1)

Table 4-1: Relationships Between Weibuil Parameters

II. Computational Methods

II.1 Linear Regression Analysis

C

As explained earlier, linear regression analysis is used to fit a straight line to the data in the various Weibull plots. Following is the code performing this analysis.

VARIABLE DECLARATIONS REAL DELTA ! ARRAY OF INTERARRIVAL TIMES DIMENSION DELTA(1000) INTEGER KSIZE ! COUNT OF INTERARRIVAL TIMES DOUBLE PRECISION SIZE 1 FLOATING VALUE OF KSIZE DOUBLE PRECISION X ! Ln OF INTERARRIYAL TIMES DIMENSION X(1000) DOUBLE PRECISION SUMX ! SUM OF X(I) DOUBLE PRECISION SUMX2 1 SUM DF X(1)**2 DOUBLE PRECISION Y ! TRANSFORMED WEIBULL COF VALUES DIMENSION Y(1000) ! SUM OF Y(I) DOUBLE PRECISION SUMY DOUBLE PRECISION SUMXY ! SUM OF X(I) * Y(I) DOUBLE PRECISION A ! SLOPE OF WEIBULL LINE (Y=AX+B) DOUBLE PRECISION B ! INTERCEPT OF WEIBULL LINE REAL ALPH, LAMBD ! LINEAR ESTIMATES DOUBLE PRECISION EPSILM ! ALLOWED ERROR IN CONVERGING DOUBLE PRECISION SUMA ! SUM OF DELTA(I) **ALPHA DOUBLE PRECISION SUMB ! SUM OF DELTA(I)**ALPHA*X(I) DOUBLE PRECISION SUMC ! SUM OF DELTA(I) **ALPHA*X(I) *** DOUBLE PRECISION DIEMP ! WORKING (TEMPORARY) VARIABLE DOUBLE PRECISION ALPHA ! MLE WEIBULL ALPHA DOUBLE PRECISION LAMBDA ! MLE HEIBULL LAMBDA

```
C
C
    SORT THE INPUT DATA
1600
        CALL SORT(DELTA, KSIZE)
C
        SIZE = DBLE( FLOAT( KSIZE ) )
        SUMX = 0.0D0
        SUMX2 = 0.0D0
        SUMY = 0.000
        SUMXY = 0.000
C
        DO 2400 I=1,KSIZE
C
C
                 PLOT Ln OF DATA ON X-AXIS
C
          X(I) = DLOG(DBLE(DELTA(I)))
C
C
                 PLOT Ln Ln (1 / (1 - F(t))) ON Y-AXIS
C
                 F(t) IS ESTIMATED BY
C
                        F(t[i]) = (i - 0.5) / NUMBER OF FAILURES
C
          Y(I) = 1.000 / (1.000 - ((DBLE(FLOAT(I))-0.500)/SIZE))
          Y(I) = DLOG(DLOG(Y(I)))
C
C
                 TRY LINEAR APPROXIMATION USING LEAST SQUARES FIT
C
                        Y = A * X + B
          SUMX = SUMX + X(I)
          SUMX2 = SUMX2 + (X(I) ** 2)
          SUMY = SUMY + Y(I)
          SUMXY = SUMXY + (X(I) * Y(I))
C
2400
          CONTINUE
        A = ((SIZE*SUMXY)-(SUMX*SUMY)) / ((SIZE*SUMX2)-(SUMX***2))
        B = (SUMY/SIZE) - (A * (SUMX/SIZE))
        ALPH = SNGL(A)
                                        I A = ALPHA
        LAMB = SNGL( DEXP(B/A) )
                                        ! B = ALPHA + Ln(LAMBDA)
```

II.2 MLE Parameter Calculation

The following code segment calculates the MLE values for the Weibull parameters. This follows the previous example in the same program.

```
C
   CALCULATE THE MAXIMUM LIKELIHOOD ESTIMATES OF ALPHA & LAMBOA
C
C
        A NEWTON-RAPHSON ITERATION METHOD IS USED TO CALCULATE
C
         ALPHA. (SEE Thoman, et al., "Inferences on the
C
         Parameters of the Weibull Distribution", TECHNOMETRICS.
C
         August 1969, page 458,)
C
C
        THE FORMULA FOR LAMBDA COMES FROM PAGE 445 OF THE SAME
C
         PAPER.
        EPSILN = 1.0D-6
        ALPHA = A
        SUMA = 0.0D0
2700
        SUMB = 0.000
        SUMC = 0.000
        DO 2800 I=1,KSIZE
          DTEMP = DEXP(X(I) * ALPHA)
          SUMA = SUMA + DTEMP
          SUMB = SUMB + (DTEMP \star X(I))
          SUMC = SUMC + (DTEMP * (X(1)**2))
2800
          CONTINUE
        DTEMP = ALPHA
        ALPHA = DTEMP +
     2
                 (((1/DTEMP)+(SUMX/SIZE)-(SUMB/SUMA)) /
                 ((1/DTEMP**2)+(((SUMA*SUMC)-SUMB**2)/SUMA**2)))
        IF ((DABS(ALPHA-DTEMP)/DTEMP) .GT. EPSILN) 2700,2000
2900
        SUMA = 0.0D0
        DO 3000 I = 1, KSIZE
          DTEMP = DEXP(X(1) * ALPHA)
          SUMA = SUMA + DTEMP
3000
          CONTINUE
        LAMBDA = DEXP( DLOG(SIZE/SUMA) / ALPHA )
```

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